The relation between the emissivity and the absorption capacity of the bodies for heat and light; by Gustav Robert Kirchhoff.[1]

Ueber das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht

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A body which is in an envelope whose temperature is equal to its own, does not change its temperature by heat radiation, and absorbs in a certain time just as many rays as it emits. It has long ago been concluded that at the same temperature, the ratio between the emissivity and the absorbency is the same for all bodies. It has been assumed that the bodies only emit rays of 'one' species. This theorem has been confirmed in many cases by experiments, especially by the de Provostaye and Desains, in which the similarity of the rays emitted could at least approximately be presupposed insofar as the rays were dark. Whether a similar proposition applies, if the bodies at the same time emit rays of different genera, which, strictly speaking, is always the case, has not yet been determined either by theoretical considerations or by experiments. I have now found that that proposition retains its validity as soon as one understands the intensity of the emitted rays of *one genus* only under the emanation ability, and obtains the absorbance of rays of the same genus. *The bond between the emissivity and the absorbency*, these terms taken in the manner indicated, *is the same for all bodies at the same temperature*. I will hereby give the theoretical proof of this proposition, and then develop some remarkable inferences which flow directly from it, and explain the partly known phenomena, and partly teach new phenomena.

Each body emits rays whose quality and intensity are dependent on its nature and temperature. To these may be added, in certain circumstances, other rays; it does so, for example, when the body is electrified to a sufficient degree, or when it phosphoresces or fluoresces. Such cases should be excluded here. If the body is hit by rays from the outside, it absorbs a part of it and transforms it into heat. For this absorption, under certain conditions, there may be another, for example, when the body is a vacuum, or when it fluoresces. It is assumed here that all absorbed rays are transformed into heat.

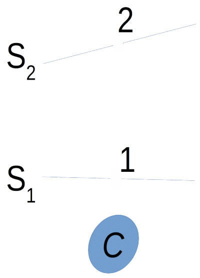

Figure 1

§. 1. Before a body *C*, Fig. 1, think of two screens S_1, and S_2, in which the two openings 1 and 2 are located, the dimensions of which are infinitesimal to their distance, and one of which has a midpoint. Through these openings occurs from the body *C* a beam. Let us consider the part whose wavelengths lie between λ and $\lambda+d\lambda$, and decompose it into two polarized components whose planes of polarization are the mutually perpendicular planes *a* and *b* passing through the axis of the beam. The intensity of the components polarized into *a*, say $Ed\lambda$: *E* gives the emissivity of the body.

On the body *C*, conversely, through the openings 2 and 1, a beam bundle of the wavelength λ polarized according to the plane *a* falls; from this the body absorbs a part, while it partly transmits

1 Also translated by Guthrie, F. as Kirchhoff, G. (1860). "On the relation between the radiating and absorbing powers of different bodies for light and heat". Philosophical Magazine. Series 4. 20: 1–21.

the rest, partly reflects it; the proportion of the intensity of the absorbed rays to that of the conspicuous, say A be the absorption capacity of the body.

The quantities E and A depend on the nature and temperature of the body C, on the position and shape of the openings 1 and 2, on the wavelength λ and the direction of the plane a. It is to be proved that the relation of E to A is independent of the nature of the body; it will be obvious that this relation is not changeable with the direction of the plane a, and its dependence on the position and shape of the openings 1 and 2 will be easily expressed, leaving only unknown how it depends on the temperature and the wavelength λ.

The proof to be given here for the assertion is based on the assumption that bodies are conceivable which, at infinitely small thickness, completely absorb all the rays which fall upon them, and thus neither reflect nor let rays pass. I want to call such bodies *ideal black*, or just *black*. It is necessary first to examine the radiation of such black bodies.

§. 2. Let C be a black body; its emissivity, generally denoted by E, is called e; it shall be proved that e remains unchanged when C is replaced by some other black body of the same temperature.

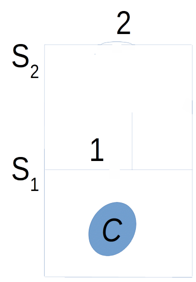

Figure 2

Imagine the body C enclosed in a black shell, of which the screen S_1 makes up a part; the second screen, like the first, is formed of a black substance, and both are connected with each other by black side walls, Fig. 2. The opening 2 is first thought of by an equally black surface, which I shall call Area 2, locked. The whole system should have the same temperature and be protected from heat loss by a heat-impermeable envelope, such as a closed, perfectly reflecting surface. The temperature of the body C then remains the same, and therefore the sum of the intensities of the rays which strike it (and which it completely absorbs on the assumption) must be equal to the sum of the intensities of the rays which it emits. Imagine that the surface 2 is removed and the exposed opening is closed by a piece of a perfectly reflecting spherical surface, directly behind it, which has its centre in the centre of the opening 1. The equilibrium of the temperature will persist even then. That equality of the intensity of the rays emitted by the body C and that of which it is hit, must therefore take place now. But since the body C now emits the same rays as in the previously imaginary case, it follows that the intensity of the rays which strike the body C in both cases is the same. By the removal of the surface 2, the bodies C are deprived of the rays which transmitted through the opening 1; for this purpose, the concave mirror attached to the opening 2 throws those rays back to the body C which the latter transmits through the openings 1 and 2[2]. It follows from this that the intensity of the beam, which the body C sends out through the openings 1 and 2, is equal to the intensity of the beam which emits the black area 2 at the same temperature through the opening 1. That intensity is therefore independent of the shape and further nature of the black body C. The pronounced proposition would be proved here if all the rays of the two rays just compared with

2 It is neglected in this case, the bending experienced by the rays at the edges of the opening 2; this is justified if one considers the openings 1 and 2, which are infinitely small against their distance but very big against the wavelength of the rays.

each other were polarized bundle by the wavelength λ and by the plane a. The consideration of the diversity of these rays makes somewhat more complicated considerations necessary.

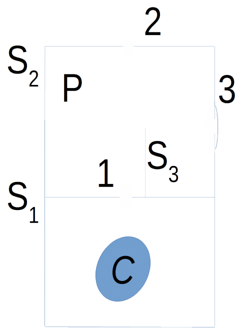

Figure 3

§. 3. In the arrangement shown in Fig. 2 one imagines between the openings 1 and 2 a lamination of thin plates P (Fig. 3) following the theory of colours from thin plates and partly because of their narrowness, partly because of its insubstantial nature, P neither emits nor absorbs a noticeable amount of radiation. The plate is directed in such a way that the beam passing through the openings 1 and 2 hits it at the angle of polarization and the plane of incidence of the plane a. The wall connecting the screens S_1 and S_2 is designed so that the mirror image which forms the panel P from the opening 2 lies in it; In the place and in the shape of this mirror image one thinks of an opening which I want to call the opening 3. The opening 2 is closed by a black surface of the temperature of the whole system, the opening 3 'once' by a surface of this kind, which I shall call the surface 3, 'the other time' closed by a perfect concave mirror, which is its centre has at the location of the mirror image that designs the plate P from the center of the opening 1. In both cases, a balance of heat takes place; from a consideration as carried out in the previous section, it follows that the sum of the intensities of the rays which are removed by removing the surface 3 from the body C is equal to the sum of the intensities of the rays created by the concave mirror be supplied. A black screen S_3 (of the temperature of the whole system) is placed so that none of the rays which the surface 3 emits directly hit the opening 1. The first sum is then the intensity of the rays which emanated from the surface 3, reflected on the plate P, and passed through the opening 1; it is called Q. The second sum is composed of two parts; the one part derives from the body C, and is:

$$=\int_0^\infty d\lambda e r^2,$$

where r signifies a quantity dependent on the nature of the plate P and the wavelength λ; the second part is due to rays which have proceeded from a part of the black wall connecting the screens S_1 and S_2, having penetrated the plate P, being reflected from the concave mirror and then from the plate P; this part is denoted by R. It is unnecessary to examine the value of R more closely; suffice it to say that R, like Q, is independent of the nature of the body C. Between the introduced quantities is the equation:

$$\int_0^\infty d\lambda e r^2 + R = Q.$$

Now, supposing that the body C is replaced by another black body of the same temperature, and for this one denotes e', which has been designated as e, then the equation:

$$\int_0^\infty d\lambda e' r^2 + R = Q.$$

From this follows:

$$\int_0^\infty d\lambda (e - e') r^2 = 0.$$

Now suppose that the refractive ratio of plate P is infinitely large compared to the unit. From the theory of the colours of thin plates it follows that

$$r=\rho\sin^2\frac{p}{\lambda}$$

can be set, where p means a size proportional to the thickness of the plate P, independent of λ, ρ means a size independent of this thickness. This becomes the derived equation:

$$\int_0^\infty d\lambda(e-e')\rho^2\sin^4\frac{p}{\lambda}=0.$$

From the fact that this equation must exist for each thickness of the plate P, as for each value of p, it can be concluded that for each value of λ

$$(e-e')=0.$$

To prove this, put it in that equation for $\sin^4\frac{p}{\lambda}$:

$$\frac{1}{8}\left(\cos 4\frac{p}{\lambda}-4\cos 2\frac{p}{\lambda}+3\right)$$

and differentiate it twice with respect to p to obtain:

$$\int_0^\infty d\lambda\frac{(e-e')\rho^2}{\lambda^2}\left(\cos 4\frac{p}{\lambda}-\cos 2\frac{p}{\lambda}\right)=0.$$

Instead of λ, introduce a new quantity α through the equation:

$$\frac{2}{\lambda}=\alpha$$

and set:

$$(e-e')\rho^2=f(\alpha).$$

This gives you:

$$\int_0^\infty d\alpha f(\alpha)(\cos 2p\alpha-\cos p\alpha).$$

Considering that if $\phi(\alpha)$ is an arbitrary function of α,

$$\int_0^\infty \phi(\alpha)\cos 2p\alpha=\frac{1}{2}\int_0^\infty d\alpha\phi\left(\frac{\alpha}{2}\right)\cos p\alpha,$$

which one can convince oneself for α put α/2, we can write for it:

$$\int_0^\infty d\alpha\left(f\left(\frac{\alpha}{2}\right)-2f(\alpha)\right)\cos p\alpha=0.$$

Multiply this equation by

$$dp\cos xp,$$

where x is an arbitrary variable, and integrate it from $p = 0$ to $p = \infty$. By Fourier's theorem, which is expressed by the equation

$$\int_0^\infty dp\cos px\int_0^\infty d\alpha\,\phi(\alpha)\cos p\alpha=\frac{\pi}{2}\phi(\lambda)$$

one obtains:

$$f\left(\frac{x}{2}\right)=2f(x)$$

or

$$f\left(\frac{\alpha}{2}\right)=2f(\alpha).$$

From this it follows that $f(\alpha)$ either vanishes for all values of α or becomes infinitely large when α approaches zero, when α approaches zero λ becomes infinite. Remembering the meaning of $f(\alpha)$, consider that ρ is a proper fraction [always positive real], and that neither e nor e' can become infinite if λ grows infinitely, one sees that the second case can not take place and therefore for all values of λ

$$e=e'$$

must be the case.

§. 4. If the bundle of rays which the black body C transmits through the openings 1 and 2 were partly polarized rectilinearly, then the plane of polarization of the polarized particle should rotate as the body C were turned about the axis of the ray-bundle. Such a rotation would therefore have to change the value of e. Since, according to the proved equation, such a change can not occur, the bundle of rays does not have a rectilinearly polarized part. It can be proved that the same thing can not have a circularly polarized part. However, the proof should not be given here. Even without them, it is admitted that black bodies can be imagined, in the structure of which there is no reason why they should in some direction send out more circularly polarized rays of the 'one' kind than circularly polarized rays of the 'other' kind. From this texture. the black bodies, which appear in further consideration, should be presupposed; they emit unpolarized rays in all directions.

§. 5. The size indicated by e depends on the temperature and the wavelength of the shape and relative position of the openings 1 and 2. If we denote by w_1 and w_2 the projections of the openings on planes that are perpendicular to the axis of the considered bundle of rays, and calls s the removal of the openings, then:

$$e = J \frac{w_1 w_2}{s^2},$$

where J only means a function of wavelength and temperature.

§. 6. Since the shape of the body C is an arbitrary one, we can substitute for it also a surface which just fills the opening 1, and which I shall call surface 1, of which the screen S_1 can then be considered. Also, the screen S_2 can be considered, if the ray bundle to which e refers is defined as that which falls from the face 1 to the face 2, which just fills the aperture 2.

§. 7. An inference from the last equation which immediately presents itself and which will be used later, is that the value of e remains unchanged when one considers the openings 1 and 2 interchanged.

§. 8. A theorem should now be proved, which can be considered as a generalization of the theorem declared in the preceding section.

Between the two black surfaces of the same temperature, 1 and 2, imagine bodies which refract, reflect, and absorb the rays which they send to each other in any way. Several beams can reach from surface 1 to surface 2; among these, choose one, consider from it the part whose wavelengths lie between λ and $\lambda + d\lambda$, and decompose it into two components whose planes of polarization are the mutually perpendicular (otherwise arbitrary) planes a_1 and b_1 are. What arrives from the first component in 2 is decomposed into two components whose planes of polarization are the mutually perpendicular (otherwise arbitrary) planes a_2 and b_2. The intensity of the components polarized according to a_2 $K d\lambda$. From the ray bundle, which goes from 2 to 1 in the same way as the previous one, consider at 2 the part whose wavelengths lie between λ and $\lambda + d\lambda$, and decompose this into two components polarized to a_2 and b_2. What arrives from the first component in 1 is decomposed into two components whose polarization planes are a_1 and b_2. The intensity of the components polarized after a_1 are $K' d\lambda$. Then

$$K = K'.$$

The proof of this theorem is first to be given on the premise that the rays in question do not suffer any weakening on their way, under the presupposition that the refractions and reflections occur without loss, that absorption does not take place, and that the polarization from 1 to a_1 emerging rays in 2 arrive at a_2 polarized, as well as vice versa.

Through the center of 1 place a plane perpendicular to the axis of the beam emerging or arriving here, and think of it as a rectangular coordinate system whose starting point is that center point x_1, y_1 are the coordinates of a point of the plane, Fig. 4. In the unit of the distance from this plane one thinks of a second one, parallel to it, and in this a coordinate system whose axes are parallel to that and whose starting point lies in the axis of the beam. x_3, y_3 are the coordinates of a point of this plane. Similarly, place a plane perpendicular to the axis of the beam emerging or incident through the center of 2, and introduce into it a rectangular coordinate system whose starting point is the said center. x_2, y_2 are the coordinates of a point of the plane. In the unity of the distance from this plane and its parallel one finally thinks of a fourth and in the same a coordinate system whose axes are parallel to the axes of x_2, y_2 and whose starting point lies in the axis of the beam. x_4, y_4 are the coordinates of a point on this fourth level.

From any point (x_1, y_1), a ray goes to any point (x_2, y_2); the time it takes to get from that point to this is T; it is a function of x_1, y_1, x_2, y_2 which should be assumed to be known. If the points (x_3, y_3) and (x_4, y_4) lie on the path of said beam, then (if for brevity the propagation velocity of the beam in empty space is assumed to be unity) the time the beam takes from (x_3, y_3) to (x_4, y_4) is

$$=T-\sqrt{1+(x_1-x_3)^2+(y_1-y_3)^2}-\sqrt{1+(x_2-x_4)^2+(y_2-y_4)^2}.$$

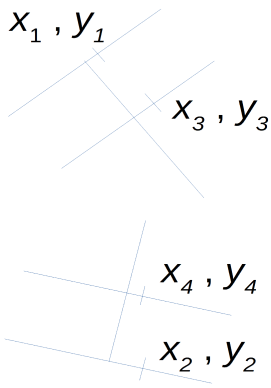

Figure 4

If the points (x_3, y_3) and (x_4, y_4) are given and the points (x_1, y_1) and (x_2, y_2) are searched for, then one would be able to find this from the condition that the expression just established is a minimum, Assuming that the 8 coordinates x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4 have infinitesimals, then the following equations express the condition that the four points (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) lie on *one* beam:

$$x_3=x_1-\frac{\partial T}{\partial x_1} \qquad x_4=x_2-\frac{\partial T}{\partial x_2}$$

$$y_3=y_1-\frac{\partial T}{\partial y_1} \qquad y_4=y_2-\frac{\partial T}{\partial y_2}.$$

Now let (x_1, y_1) be a point of the projection of surface 1 onto the plane of x_1, y_1, $dx_1\, dy_1$ an element of this projection in which the point (x_1, y_1) lies and which is infinitely small from a higher order than the surfaces 1 and 2 are. (x_3, y_3) let be a point of a ray which hits surface 2 starting from (x_1, y_1), $dx_3\, dy_3$ a surface element in which the point (x_3, y_3) lies, of the same order as dx_1, dy_1. The intensity of the rays from the designated wavelengths and the chosen direction of polarization, which pass through $dx_3\, dy_3$ starting from $dx_1\, dy_1$, is then after §5:

$$(d\lambda)\, I\, dx_1\, dy_1\, dx_3 dy_3.$$

According to the assumption made, the amount of radiation reaches the surface 2 without weakening and forms an element of the magnitude designated $Kd\lambda$. K is the well-bound integral

$$I\iiiint dx_1\, dy_1\, dx_3\, dy_3.$$

Here we have to extend the integration to x_3 and y_3 over those values which obtain these quantities according to the equations established for them, while x_1 and y_1 keep constant values and x_2, y_2 take all values that correspond to the points of the projection of surface 2 onto the plane corresponding to x_2, y_2; then the integration to x_1, y_1 over the projection of the surface 1 is to be carried out. However, the double integral

$$\iint dx_3\, dy_3$$

is

$$=\iint\left(\frac{\partial x_3}{\partial x_2}\frac{\partial y_3}{\partial y_2}-\frac{\partial x_3}{\partial y_2}\frac{\partial y_3}{\partial x_2}\right)dx_2\, dy_2$$

or according to the equations for x_3, y_3

$$=\iint\left(\frac{\partial^2 T}{\partial x_1 \partial x_2}\frac{\partial^2 T}{\partial y_1 \partial y_2}-\frac{\partial^2 T}{\partial x_1 \partial y_2}\frac{\partial^2 T}{\partial x_2 \partial y_1}\right)dx_2\, dy_2\,,$$

where the integration is to be extended beyond the surface 2 projection. Hereinafter:

$$K=\int\int\int\int\left(\frac{\partial^2 T}{\partial x_1 \partial x_2}\frac{\partial^2 T}{\partial y_1 \partial y_2}-\frac{\partial^2 T}{\partial x_1 \partial y_2}\frac{\partial^2 T}{\partial x_2 \partial y_1}\right)dx_1\, dy_1\, dx_2\, dy_2\,,$$

where the integration is to be taken over the projections of surfaces 1 and 2.

If we treat in the same way the quantity denoted by K', using a ray of the same time to travel the distance between two points in one sense or the other, we find for K' the same expression as for K is found.

In this way the pronounced proposition is proved under the restricting presupposition under which it should first be proved. However, this limitation is directly raised by a remark made by Helmholtz in his Physiological Optics p. 169. Helmholtz says (using names defined here): “A ray of light comes after any number of refractions, reflections, and so on, from the point 1 to the point 2 In point 1, one places by its direction two arbitrary mutually perpendicular planes a_1 and b_1, according to which its vibrations are thought of decomposed. Two such planes a_2 and b_2 are placed in point 2 by the beam. Then the following can be proved: If the quantity i emanates from the plane a_1 of polarized light of 1 in the direction of the beam in question, and from this the quantity k arrives at the plane a_2 of polarized light in 2, then if quantity i emanates from 2 according to a_2 polarized light, the same quantity k arrives at 1 from a_2 polarized light[3].”

Using this theorem, denoting by γ the value of the ratio k/i for the two rays moving between the points (x_1, y_1) and (x_2, y_2) in one and the other sense, so we obtain for both K and K' an expression which differs from the one found only in that γ occurs among the integral signs as a factor.

The equality of K and K' also takes place thereafter, when gamma has a different value for the ray into which one of the compared ray bundles can be tapped; for example, it does not stop when a part of the beam is caught by a screen.

§. 9. From the same bundles of rays, which are compared with each other in the previous section, the following proposition holds: From the bundle of rays which goes from 1 to 2, consider at 2 the part whose wavelengths lie between λ and $\lambda + d\lambda$, and decompose it into two components polarized according to a_2 and b_2: the intensity of the first component of $Hd\lambda$. From the ray bundle which goes from 2 to 1, consider at 2 the part whose wavelengths lie between λ and $\lambda + d\lambda$, and decompose it into 2 components polarized to a_2 and b_2. What arrives from the first component in 1 is $H'd\lambda$. Then

3 As Helmholtz notes, Helmholtz's theorem does not apply when the plane of polarization of the ray undergoes a turn, as magnetic forces produce after the discovery of Faraday; in the following considerations, therefore, one has magnetic powers to think themselves ineffective. Helmholtz also limits his theorem by assuming that the light does not undergo a change in the refractive power, as occurs in fluorescence; this limitation ceases to be necessary, if in the application of the theorem one only considers rays of 'one' wavelength in the eye.

$$H = H'.$$

The proof of this theorem is the following. K and K' should have the same meaning as in the previous section; L and L' are the quantities that arise from K and K' when the plane a_1 is swapped with the plane b_1. Then $L = L'$, as well as $K = K'$. Further,

$$H = K - L,$$

because perpendicularly polarized rays do not interfere, if they are reduced to a common plane of polarization, if they are parts of an unpolarized ray, and according to §4, the surface 1 emits non-polarized rays.

Finally, one has

$$H' = K' + L',$$

because two rays whose polarization planes are perpendicular to each other do not interfere. From these equations follows: $H = H'$.

§.10. In Figure 2, we have the same meaning given in §3, except that the body C is not a black, but an arbitrary one. Let the opening 2 be closed in the surface 2. This surface transmits through the opening 1 on the body C a bundle of rays, which is partly absorbed by it, and scattered in part by refractions and reflections in different directions. From this ray bundle, consider between 2 and 1 the part whose wavelengths lie between λ and $\lambda + d\lambda$, and decompose it into two components polarized according to the plane a and the plane perpendicular thereto. What gives the first component of absorption by the body C, that is, the black shell in which the body C is enclosed, is $M'd\lambda$. Of the rays which send the parts of this covering to the body C, certain will fall through the opening 1 on the surface 2; as a result of the mediation of the body C, a bundle of rays is produced which passes through the opening 1 towards the surface 2. From this consider the part whose wavelengths lie between $\lambda + d\lambda$, and decompose it into two components polarized according to a and the plane perpendicular to a. The intensity of the first component is $Md\lambda$. Then

$$M = M'.$$

The correctness of this proposition results from the proposition of the previous section, if one applies it to all the rays which exchange the surface 2 and one element each of the black envelope which surrounds the body C, by the agency of the body C; and then form the sum of the equations that you get.

§. 11. Consider the arrangement shown in Figure 3 and described in §3; only the body C is not a black, but an arbitrary one. In the two cases described there, the equilibrium of the heat takes place; Even then, therefore, the active energy, which is removed by removing the black surface 3 of the body C must be equal to the active energy, which is supplied to this by mounting the concave mirror. The designations used in §3 are to be used here unaltered in meaning; the characters E and A should have the definitions given in §1. The active energy, which is removed from the body C by the removal of the surface 3, is in consideration of §7

$$= \int_0^\infty d\lambda \; e r A.$$

The active energy, which the body C gains through the mediation of the concave mirror, consists of three parts together. The first of these parts stems from rays emitted by the body C itself; it is

$$= \int_0^\infty d\lambda \; E r^2 A.$$

The second part is due to rays emanating from the black wall opposite the concave mirror, penetrating the plate P, one at the concave mirror, and a second at the plate P. Have suffered reflection; it is after §9:

$$= \int_0^\infty d\lambda \; e r (1 - r) A.$$

The third part, at last, stems from rays which fall upon it from various points of the black envelope which surrounds the body *C*, thrown from it through the opening 1 to the surface 2 by reflections or refractions, and by a reflection on the plate P, a second on the concave mirror and a third are driven back to the plate P through the opening 1. If we use the *M* defined in the §10, then this part is

$$=\int_0^\infty d\lambda \;\; M r^2 A\,.$$

It may seem doubtful whether the first and the third are correctly indicated, if the body *C* has such a position, that a finite part of the bundle of rays, which sends the surface 2 through the opening 1, is thrown back from it to the surface 2. Such cases are therefore provisionally excluded.

By §10 we have $M = M'$, and according to the definition of M', we have

$$M' = e(1-A).$$

That third part is therefore

$$=\int_0^\infty d\lambda \;\; e(1-A) r^2 A,$$

and the equation is given:

$$\int_0^\infty d\lambda \;\; (E-Ae)\, A r^2 = 0\,.$$

By the same considerations made in §3 with respect to a similar equation, one gets from this the conclusion that for every value of λ

$$\frac{E}{A} = e\,,$$

or, if one sets its value for *e* from §5:

$$\frac{E}{A} = J\,\frac{w_1 w_2}{s^2}\,.$$

In this way the proposition which ought to be proved in this treatise is proved on the supposition that of the bundle of rays falling from the surface 2 through the opening 1 upon the body *C*, no finite part is thrown back by this to the surface 2; that the theorem holds without this limitation, one sees, when one considers that, if the said condition is not fulfilled, one needs only to turn the body *C* infinitely little to satisfy it, and that by such a rotation the quantities *E* and *A* can only undergo infinitely small changes.

The quantity designated *J* is, as noted in §5, a function of wavelength and temperature. It is a task of great importance to find this function. The experimental determination of the same poses great difficulties; nevertheless, the hope seems to be able to determine it by experiments, since it is undoubtedly of simple form, as are all functions which do not depend on the properties of individual bodies, and which have hitherto been known. Only when this task is solved will the whole fruitfulness of the proved proposition be revealed; but even now important conclusions can be drawn from it.

§. 12. If, for example, a certain body, a platinum wire, is gradually heated, it radiates, until its temperature has become a certain one, only rays whose wavelengths are greater than those of the visible rays. At a certain temperature rays begin to show the wavelength of the extreme red; If the temperature is higher and higher, then rays of smaller and smaller wavelengths are added, such that at any temperature rays of a corresponding wavelength occur while the intensity of the rays of longer wavelengths grows. Applying the proved theorem to this case, one sees that the function *J*, for a wavelength equal to zero, grows for all temperatures below a certain temperature corresponding to the wavelength and for higher temperatures therewith. Hence, applying the same proposition to other bodies, it follows that all bodies, when their temperature is gradually increased, begin to emit rays of the same wavelength at the same temperature, that is, to glow red at the same

temperature, at a higher, all common Start outputting temperature yellow rays and so on[4]. The intensity of the rays of a certain wavelength, which different bodies send out at the same temperature, may, however, be very different; it is proportional to the absorption rates of the bodies for rays of the wavelength in question. At the same temperature, therefore, metal glows more vividly than glass, and this more than a gas. A body that would remain completely transparent at the highest temperatures would never glow. In a ring, about 5 mm in diameter, bent from platinum wire, I brought a little sodium hydrogen phosphate, and heated it in the little luminous flame of Bunsen's gas-lamp. The salt melted, forming a liquid lens, remaining perfectly clear; but it did not shine at all, while the same touching platinum ring exuded the most vivid light.

§. 13. For a constant temperature, the function *J* changes continuously with the wavelength, as long as it does not receive the value at which *J* begins to disappear for that temperature. The correctness of this assertion can be deduced from the continuity of the spectrum of a glowing platinum wire, as soon as one assumes that the absorption capacity of this body is a continuous function of the wavelength of the incident rays. It can also be said with the highest degree of probability that the function *J* does not show strong peaks or minima when the temperature remains constant, as the wavelength changes. It follows that if in the spectrum of a glowing body there are cracks, strongly protruding maxima or minima, the absorption capacity of the same, considered as a function of the wavelength of the incident rays, must have cracks, strongly prominent maxima or minima at the same values of the wavelength. Spectra with very striking maxima are obtained by bringing certain salts into the flame of Bunsen's lamp. A particularly interesting example of this is provided by the lithium chloride. When a globule of this salt is melted on a platinum wire, and brought into the mantle of the gas-flame, the spectrum of the flame (unless it is contaminated by other salts and the intensity of light is not increased too much) is shown to be a single brilliant red line whose wavelength is approximately the arithmetic mean of the wavelengths corresponding to Fraunhofer's lines B and C For this wavelength, the emissivity of the flame has great value, while for all other visible wavelengths, it is vanishingly small. Accordingly, the absorptivity of the lithium flame must have a considerable value for the same wavelength, but imperceptibly for all other visible rays. Therefore, if one forms a continuous spectrum from a suitable light source, and brings a lithium flame between them and the gap necessary for it, it changes the brightness in the spectrum 'only' at the location of the lithium line. Here it increases the brightness by its own light, but weakens it by the absorption that it exerts on the rays of the corresponding wavelength passing through it. Let this absorption be a quarter. It would, according to the proved proposition, be the case if the brightness of the bright line of the lithium flame were one fourth of the brightness at the same place of the spectrum, which in the same apparatus is a perfectly black body from the temperature of the flame would give. Unchanged then the lithium flame would leave the brightness at the considered location of the spectrum if the brightness of the lithium line, while the lights of the rear light source are dimmed, is a quarter of the brightness which takes place at the same place of the spectrum while the rear light source works alone. If this light source has a greater brightness than that determined by it, the lithium line turns out to be darker on lighter ground, and in the opposite case bright on darker ground. If the first takes place, the greater the intensity of the dark source, the darker the line will be; because the more this light intensity is increased, the more imperceptible becomes the own light of the lithium flame. Meanwhile, with the numerical value assumed for the absorption of the latter, the brightness of the line can never fall below three-quarters of the brightness of the environment. This limit, however, is suppressed by increasing the thickness of the lithium flame, and thus its absorption.

A small globule of lithium chloride brought into the flame of the Bunsen lamp gives it already such a great power of absorption for the rays of the designated wavelength, that if one lets sunbeams pass

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4 Draper, Phil. Mag. XXX, p. 345; 1847. On the production of heat and light by John William Draper, Journal of the Franklin Institute Volume 44, Issue 2, August 1847, Pages 122-128

through the flame onto the gap of the apparatus which forms the spectrum the corresponding place a black appearing, fine line shows itself.

The spectra, which cause other salts when brought into the flame, are usually less simple and rarely present lines that are as bright as the lithium line. But all these spectra must be reversed in a similar way; if one gives the flame sufficient thickness and allows light of sufficient intensity to pass through it, the previously bright lines must pass into dark ones. An exception could only arise with a flame in which a part of the light would be produced directly by chemical process, or by a flame which fluoresced. The experiments must decide whether there are such flames.

If the rear light source is a glowing body, its intensity depends on the temperature of the latter; the intensity has its highest value at a given temperature when the body is completely black. If this condition is fulfilled and the temperature of the two light sources is the same, the front just leaves the spectrum of the rear unchanged. The rear light source, therefore, can only reverse the spectrum of the front if it has a higher temperature than this, and the reverse spectrum will have the greater the greater the difference in temperature between the two light sources.

So far, we have succeeded in reversing the spectra on the lithium flame, or on the common salt flame. As is known, the spectrum of these consists of two very close, shiny, yellow lines whose wavelengths coincide with the wavelengths of the two Fraunhofer lines D. If one lets the rays of the Drummond's light through a saline flame of not too high temperature, then the bright lines turn into dark ones, which are thus at the place of the Fraunhofer lines D, and which in every respect the same sight, like this, grant.

§. 14. As will be explained in detail in another place, the wavelengths for which maxima of emissivity and absorbency take place are independent of the temperature in the farthest limits; furthermore, in the salts which produce such maxima in a flame, the 'metals' which cause them. Imagine a body of very high temperature, in whose spectrum the dark double line D does not appear, surrounded by: a gaseous atmosphere of slightly lower temperature. If there is sodium in this, the dark double line D can appear in the spectrum of the light source thus composed; from the existence of these must be inferred to the sodium content of the atmosphere. Now the sun is undoubtedly is a body of a kind that is subject to speculation,[5] but from the line D of the solar spectrum is at least to close on the sodium content of the solar atmosphere.

One objection can be raised against the correctness of this conclusion: it could be in the atmosphere of the Earth the cause of the line D must be sought. This objection, however, is refuted by the following reasons;

1. The sufficient quantity of sodium in vapor form may not be well present in our atmosphere, and the vapor form would necessarily produce the effect in question;

2. If the line D originated from our atmosphere, it would have to become clearer in the measure in which the sun approaches the horizon; but I have never perceived the corresponding changes in their distinctiveness, whereas they have often been very striking to me, especially in neighboring lines;

3. If the line D did not have its ground in the Sun itself, then it would also have to be found in the spectra of all fixed stars that are bright enough; According to the data of Fraunhofer and Brewster, however, it is missing in the spectra of some fixed stars, while it is present in those of others.

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5 The question of whether the inner part of the sun, from which the light mainly radiates, is solid, liquid, or gaseous, may here be regarded as an open one.

The exact coincidence of the sodium lines with the Fraunhofer lines D can be proved most reliably if the sun's rays are allowed to pass through a sodium flame before they reach the gap of the apparatus. The effect of the sodium flame is then shown by the fact that the lines D are much clearer, blacker and wider. It has at the first moment something puzzling that the sodium in the small flame can still markedly enhance the effect which the sodium has exerted on the rays of light in the tremendous solar atmosphere. But the disconcerting part disappears when one considers that the brightness of the lines D in the solar spectrum is conditioned by the temperature of the solar atmosphere, especially its superficial layers, and that the temperature of this is certainly much greater than that of a fluorescent gas flame. If one thinks of a sodium flame, the thickness of which can be considered infinite with respect to the absorption of the rays corresponding to the lines D, and supposes that they pass the rays of a rear light source and then be separated into a spectrum, then the brightness at the locations of the lines D depends solely on the temperature of that flame. If one puts before this one sodium flame of the same temperature, this changes nothing in the spectrum; but if the added flame has a lower temperature, it must make the lines D appear darker. The effect of the luminous gas flame, into which sodium is brought, on the solar spectrum, is explained hereafter, as soon as it is added, that its temperature is lower than the temperature of the outermost layers of the solar atmosphere; but this is certainly the case, since the outermost layers of the solar atmosphere can not have a lower temperature than that which takes place in the focal point of a sun-directed, very effective concave mirror.

Similar things, like sodium, apply to any other substance which, when brought into a flame, causes bright lines to appear in its spectrum. If these lines coincide with darkening of the sunspectrum, one must deduce the presence of this substance in the solar atmosphere, provided that the dark lines in question can not have their origin in the earth's atmosphere. Thus, a way is found to determine the chemical nature of the solar atmosphere, and the same way promises some information about the chemical nature of the lighter fixed stars.[6]

§. 15. From the proposition which is proved in the first part of this treatise, it follows that a body which absorbs more of rays of 'one' direction of polarization than of those of 'another,' emits more rays in the same proportion from the first direction of polarization, as of those of the second. Thereafter, as is well known, a glowing, opaque body, having a smooth surface, must emit light in directions which are oblique to this surface, which is partly polarized, perpendicular to the plane passing through the ray and the normal of the surface goes; for of incident rays, which are polarized perpendicular to the plane of incidence, the body reflects less, so absorbs more than rays whose plane of polarization is the plane of incidence. One can easily state the polarization state of the emitted rays according to that law, if one knows the law of the reflection of incident rays.

A tourmaline plate ground parallel to the optical axis absorbs at ordinary temperature rays perpendicular to it, more when the polarization plane thereof is parallel to the axis, than when the plane of polarization is perpendicular to the axis. Provided that the tourmaline plate retains this property in incandescent heat, it must radiate with it, in a direction perpendicular to it, rays which are partly polarized, in the plane laid by the optical axis, in a plane perpendicular to that plane which is called the polarization plane of tourmaline. I have tested this striking conclusion resulting from the developed theory by the experiment, and it has been confirmed. The used tourmaline plates, brought into the flame of Bunsen's lamp, for a long time sustained a moderate heat of incineration, without a permanent change; only at the corners they became clouded after cooling down. The property of polarizing light penetrated them also in the heat of reflection, though to a

6 In two of my communications, which are submitted to the Berlin Academy on Oct. 27 and Dec. 15, 1859, there are still a few statements which are not quoted here, and which are based on the chemical nature of the solar atmosphere. In the second of these communications, moreover, the proposition which forms the main content of this essay is proved in another way, but in lesser universality than here.

much lesser degree than at low temperature. It showed itself by looking through a double-refracting prism through the tourmaline plate for a platinum wire glowing in the same flame. The two images of the platinum wire had an uneven brightness, but their difference was much less than when the tourmaline plate was outside the flame. The position was given to the birefringent prism, at which the difference of the light intensity of the two images of the platinum wire was a maximum; the brighter picture would have been the upper one; Then, after removing the platinum wire, the two images of the tourmaline plate were compared with each other. It was the upper picture, not conspicuous, but undoubtedly darker than the lower one; the two pictures just appeared, as two equal, glowing bodies had appeared, of which the upper would have possessed a lower temperature than the lower one.

§. 16. Another conclusion from the proved theorem may find room here at the end. If a space is enclosed by bodies of the same temperature, and no rays can penetrate through these bodies, then each ray-bundle in the interior of the space is of its quality and intensity just as if it came from a perfectly black body of the same temperature irrespective of the nature and shape of the body and only due to the temperature. The correctness of this assertion is seen when one considers that a bundle of rays, which has the same shape and the opposite direction as the chosen one, is completely absorbed in the infinite number of reflections which it experiences on the imaginary bodies. In the interior of an opaque, glimmering body of a certain temperature, the same brightness always takes place, which may be the same in the rest.

The theorem in this section, as may be remarked casually, can not cease to be valid even if there are fluorescents among the imaginary bodies. A fluorescent body can be defined as one in which the emissivity depends on the rays that strike it at the moment in question. The equation $\frac{E}{A}=e$ can not generally hold for such a body, but it applies to it if it is enclosed in a completely black shell of the same temperature, for the same considerations by which this equation holds for that If body C is proved to be non-fluorescent, it is also believed to fluoresce. In order to see this, one must only note that if the quantity E can have two different values in the two arrangements of the system shown in Fig. 3, if the body C fluoresces, then these two values are only infinitely small can differ.
Heidelberg, January 1860.

[Lost in Translation:

Professor Kirchhoff needed a mathematical description of the medium he was using in order to describe mathematically what he was doing in the 'balance of heat' and he used 'the theory of colours of thin plates.' Now the computer translation gave 'the theory of colours of thin phamphlets' which at first seemed to be a criticism of the large tome 'Theory of Colour' by Goethe. It took a bit of digging around to find that Lord Rayleigh had given a lecture at the Royal Institution on 'The theory of colours of thin plates' which I took to be the translation into English of the German name to the mathematical description of the physics illustrated by the papers by Brewster[7] and Wilde[8].

'Ein ächter Bruch' has been given here the interpretation 'a proper fraction' given the context of the thing it refers to and why that thing needed to be described in this way. However I have included in the text the notion that is only alluded to that that thing is a 'positive finite number' so cannot be zero, negative nor infinite.

A translation the relevant paragraphs on page 169 of an 1869 edition of Helmholtz's Physiological Optics follows with Helmholtz's qualification of his 'law:'
[https://archive.org/details/handbuchderphysi00helm/page/n11]

A ray of light passes from the point A to any number of refractions, reflections, and so forth after the point B. In A let one place by his direction two real planes a_1 and a_2 perpendicular to each other, according to which his oscillations are thought decomposed. Two such planes b_1 and b_2 are placed in B by the beam. Then the following can be proved: If the quantity J is based on the plane a_1 of polarized light of A in the direction of the beam discussed, and on that the quantity K arrives at the plane b_1 of polarized light in B, then if the quantity J goes backward, becomes B_1 polarized light emanates from B, the same quantity K after a_1 polarized light arrive in A,

As far as I can see, the light may in this case be subject to simple and double refraction, reflection, absorption, ordinary dispersion and diffraction, without the law losing its applicability, only that there can be no alteration of its refrangibility, and not by body go in which the magnetism acts after Faraday's discovery on the position of the polarization plane.

And the translation carried out in 1924 [*Helmholtz, Hermann von; Southall, James Powell Cocke [ed.]*, **Treatise on physiological optics; vol. 1**, 1924]:

Suppose light proceeds by any path whatever from a point A to another point B, undergoing any number of reflexions or refractions *en route*. Consider a pair of rectangular planes a_1 and a_2 whose line of intersection is along the initial path of the ray at A; and another pair of rectangular planes b_1 and b_2 intersecting along the path of the ray when it comes to B. The components of the vibrations of the aether particles in these two pairs of planes may be imagined. Now, suppose that a certain amount of light J leaving the point A in the given direction is polarised in the plane a_1, and that of this light the amount K arrives at the point B polarised in the plane b_1; then it can be proved that, when the light returns over the same path, and the quantity of light J polarised in the plane b_1 proceeds from the point B, the amount of this light that arrives at the point A polarised in the plane a_1 will be equal to K.

7 On the Connexion between the Phenomena of the Absorption of Light and the Colours of Thin Plates. Brewster, D. Abstracts of the Papers Printed in the Philosophical Transactions of the Royal Society of London (1800-1843). 1830-01-01. 3:475–476 and On the Connexion between the Phenomena of the Absorption of Light, and the Colours of Thin Plates. Brewster, D Philosophical Transactions of the Royal Society of London (1776-1886). 1837-01-01. 127:245–252 https://doi.org/10.1098/rspl.1830.0290

8 Wilde, E. (1851). *Die Theorie der Farben dünner Blättchen. Annalen Der Physik Und Chemie, 158(1), 18–41.* doi:10.1002/andp.18511580103

Apparently the above proposition is true no matter what happens to the light in the way of single or double refraction, reflection, absorption, ordinary dispersion, and diffraction, provided there is no change of its refrangibility, and provided it does not transverse any magnetic medium that affects the position of the plane of polarisation, as Faraday found to be the case. [http://echo.mpiwg-berlin.mpg.de/ECHOdocuView?url=/mpiwg/online/permanent/library/HSHPBYTP/pageimg&start=261&pn=265&mode=imagepath]]

‘Die lebendige Kraft’ becomes ‘the active energy.’

‘phosphorsaures Natron’ becomes ‘sodium hydrogen phosphate’

I’ve left the Google translation a little rough for the demonstration of the original idea is greater value than the smoothing into fluent English which might introduce new errors. Sometimes there is a reference to ray bundles instead of beams. I’ve left this for I would want to encourage the application of newer mathematics to the physics presented.

If only:-
Now, the main standing of this paper is the introduction to the idea of ‘ideal black bodies’ as universal absorbers of light radiation. Now I am aware that the nature of Black as a characteristic has gathered many connotations that would be discriminatingly disparaging to many non-white ethnic groups. So I have taken a little time to consider what Professor Kirchhoff was trying to say and then make an Ancient Greek sounding new word to be assigned to the effect and designation. After looking in a number of Ancient Greek dictionaries for word to express ‘absence of light’ I have found that there is a couple of short words that might serve: under ‘absence’ there is μη for ‘lest’ or ‘not’ and under light φως. The prevention of returning light seems to be a **mephotic** quality. So I propose that instead of the ‘ideal black body’ there is used ‘mephotic body.’]